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LETTER TO THE EDITOR

Semiclassical prediction for shot noise in chaotic cavities

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Abstract

We show that in clean chaotic cavities the power of shot noise takes a universal form. Our predictions go beyond previous results from random-matrix theory, in covering the experimentally relevant case of few channels. Following a semiclassical approach we evaluate the contributions of quadruplets of classical trajectories to shot noise. Our approach can be extended to a variety of transport phenomena as illustrated for the crossover between symmetry classes in the presence of a weak magnetic field.

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(Some figures in this article are in colour only in the electronic version)

Ballistic chaotic cavities have universal transport properties, just as do disordered conductors. The explanation of such universality cannot rely on any disorder average but must make do with chaos in an *individual* clean cavity. We shall present here the semiclassical explanation of shot noise, relating the quantum properties of chaotic cavities to the interference between contributions of mutually close classical trajectories. Similar methods have recently been used for explaining universal spectral fluctuations of chaotic quantum systems [1, 2], and to calculate the universal mean conductance in [3, 4].

Following Landauer and Büttiker [6, 7], we treat transport as scattering between two leads attached to the cavity. One lead is assumed to support N_1 ingoing channels and the second one N_2 outgoing channels. In contrast to the random-matrix treatment of [5, 7] and work on quantum graphs in [8], our results cover all orders in the inverse number of channels, $N = N_1 + N_2$, and thus apply to the experimentally relevant case of few channels [9]. Previously unknown and surprisingly simple expressions for the shot noise arise, both with and without time reversal invariance (see equation (11) below).

We shall invoke the semiclassical limit (formally $\hbar \to 0$). On the other hand, the Ehrenfest time $T_E \sim \log \frac{\text{const}}{\hbar}$ is assumed to be much smaller than the average dwell time, which means that bringing $\hbar$ to zero must be accompanied by suitably shrinking the cavity openings. Most recent investigations of corrections connected with a finite ratio of Ehrenfest time and dwell time can be found in [10, 11].

The transition amplitudes between ingoing channels a_1 and outgoing channels a_2 define an $N_1 \times N_2$ matrix $t = \{t_{a_1 a_2}\}$. That matrix determines the power of shot noise as $P = \langle \mathrm{Tr}(tt^\dagger - tt^\dagger tt^\dagger) \rangle$, in units $\frac{2e^3|V|}{\pi\hbar}$ depending on the voltage V; for us, $\langle \cdots \rangle$ denotes an average over a small energy interval. Previous work had involved averages over ensembles of matrices t and obtained [5, 7]

$$P = \frac{N_1^2 N_2^2}{N^3} + \left(\frac{2}{\beta} - 1\right) \frac{N_1 N_2 (N_1 - N_2)^2}{N^4} + \mathcal{O}\left(\frac{1}{N}\right). \tag{1}$$

Here $\beta = 1$ refers to the so-called òrthogonal case' of time-reversal invariant dynamics; if a magnetic field is applied to break time-reversal invariance ('unitary case', $\beta = 2$), the second ('weak localization') term disappears. Higher orders in $\frac{1}{N}$ are as yet unknown, apart from the case $N_1 = N_2 = 1$ [12].

In the semiclassical limit each transition amplitude $t_{a_1 a_2}$ is given by a sum over trajectories α leading from an ingoing channel a_1 to an outgoing channel a_2, $t_{a_1 a_2} \sim \sum_{\alpha(a_1 \to a_2)} \frac{A_\alpha}{\sqrt{T_H}} \mathrm{e}^{\mathrm{i}S_\alpha/\hbar}$ [13]. All relevant trajectories have the same entrance and the same exit angles determined by the ingoing (a_1) and outgoing (a_2) channels, respectively. The phase of each contribution is proportional to the classical action S_α while the factor A_α reflects the stability of the trajectory; T_H denotes the Heisenberg time $T_H = \frac{\Omega}{(2\pi\hbar)^{f-1}}$, with Ω being the volume of the energy shell and f the number of freedoms.

With the transition amplitudes thus semiclassically approximated, the quadratic term $\langle \mathrm{Tr}(tt^\dagger) \rangle$ becomes a double sum over trajectories. That double sum, which actually is the mean conductance, was evaluated in [4] as $\frac{N_1 N_2}{N - 1 + 2/\beta}$. The quartic contribution to shot noise turns into a sum over quadruplets of trajectories,

$$\begin{aligned} \langle \mathrm{Tr}(tt^\dagger tt^\dagger) \rangle &= \sum_{\substack{a_1, c_1 \\ a_2, c_2}} t_{a_1 a_2} t^*_{c_1 a_2} t_{c_1 c_2} t^*_{a_1 c_2} \\ &= \frac{1}{T_H^2} \left\langle \sum_{\substack{a_1, c_1 \\ a_2, c_2}} \sum_{\alpha, \beta, \gamma, \delta} A_\alpha A^*_\beta A_\gamma A^*_\delta \mathrm{e}^{\mathrm{i}(S_\alpha - S_\beta + S_\gamma - S_\delta)/\hbar} \right\rangle. \end{aligned} \tag{2}$$

Here $a_1, c_1 = 1, \ldots, N_1$ and $a_2, c_2 = 1, \ldots, N_2$ represent ingoing and outgoing channels, connected by the trajectories $\alpha, \beta, \gamma, \delta$ like α $(a_1 \to a_2)$, β $(c_1 \to a_2)$, γ $(c_1 \to c_2)$, δ $(a_1 \to c_2)$. The sum is dominated by quadruplets where the trajectories β and δ have approximately the same cumulative action as α and γ, such that the action difference $\Delta S \equiv S_\alpha - S_\beta + S_\gamma - S_\delta$ is of the order $\hbar$. The contributions of other quadruplets interfere destructively.

Diagonal contribution. The simplest quadruplets have either $\alpha = \beta, \gamma = \delta$, or $\alpha = \delta, \beta = \gamma$ [8]; their action difference vanishes. The first case has coinciding ingoing channels a_1 and c_1 and contributes

$$\langle \mathrm{Tr}(tt^\dagger tt^\dagger) \rangle_{\substack{\alpha=\beta \\ \gamma=\delta}} = \frac{1}{T_H^2} \sum_{\substack{a_1 \\ a_2, c_2}} \sum_{\substack{\alpha(a_1 \to a_2) \\ \gamma(a_1 \to c_2)}} |A_\alpha|^2 |A_\gamma|^2. \tag{3}$$

The foregoing sum can be done using ergodicity. As shown in [3], summing all trajectories between two fixed channels amounts to integrating over the dwell time T,

$$\sum_{\alpha(a_1 \to a_2)} |A_\alpha|^2 = \int_0^\infty \mathrm{d}T\, \mathrm{e}^{-\frac{N}{T_H} T} = \frac{T_H}{N}, \tag{4}$$

where $\mathrm{e}^{-\frac{N}{T_H} T}$ is the probability for the trajectory to dwell in the cavity up to the time T, and $\frac{N}{T_H}$ the rate of escape.

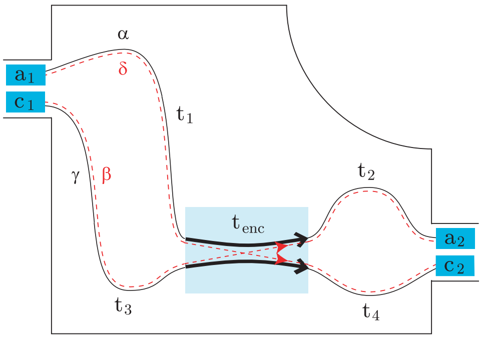

Figure 1. Quadruplet of trajectories $\alpha, \beta, \gamma, \delta$ differing by their connections inside a 2-encounter (in the box). Initial and final points marked by channel indices a_1, c_1, a_2, c_2. The durations are t_{enc} for the encounter and t_1, t_2, t_3, t_4 for the loops.

To proceed with equation (3) we invoke the Richter/Sieber sum rule (4) twice and afterwards sum over all $N_1 N_2^2$ possible combinations of channels with $a_1 = c_1$. Similarly, the case $\alpha = \delta, \beta = \gamma$ leads to $N_1^2 N_2$ combinations with coinciding outgoing channels $a_2 = c_2$. Altogether, these so-called diagonal contributions sum up to $\frac{N_1^2 N_2 + N_1 N_2^2}{N^2} = \frac{N_1 N_2}{N}$. In the unitary case, they cancel with $\langle \mathrm{Tr}(tt^\dagger) \rangle$ such that shot noise must be entirely due to different quadruplets of trajectories.

2-encounters. The first family of such quadruplets, depicted in figure 1, was identified by Schanz, Puhlmann and Geisel for quantum graphs [8]. Here, the trajectories α and γ approach each other in a '2-encounter': a stretch of α comes so close in phase space to a stretch of γ that the motion over the two stretches is mutually linearizable. The remaining parts of α and γ will be called 'loops'. Assuming that all loops have non-vanishing length (encounter stretches do not 'stick out' into the leads), one finds two further trajectories, β and δ, which practically coincide with α and γ inside the loops but are differently connected in the encounter: The trajectory β closely follows the initial loop of α and the final loop of γ, whereas δ follows the initial loop of γ and the final loop of α. Obviously, the cumulative action of β and δ approximately coincides with the action of α and γ, with the action difference exclusively determined by the encounter region.

Each encounter influences the survival probability. If a particle stays in the cavity along the first encounter stretch it cannot escape during the second stretch either, since the two stretches are close to each other. The trajectories α and γ are thus exposed to the danger of getting lost only on the four loops (see figure 1) and on *one* encounter stretch. Denoting the duration of the latter stretch by t_{enc}, we can write the overall exposure time as $T_{\text{exp}} = t_1 + t_2 + t_3 + t_4 + t_{\text{enc}}$. That exposure time is *smaller* than the cumulative duration $T_\alpha + T_\gamma$ of α and γ, by a second summand t_{enc} for the second encounter stretch. The probability that both α and γ stay inside the cavity reads $\mathrm{e}^{-\frac{N}{T_H} T_{\text{exp}}}$, larger than the naive estimate $\mathrm{e}^{-\frac{N}{T_H}(T_\alpha + T_\gamma)}$. In brief, encounters hinder escape [4].

To describe the geometry of encounters, we consider a Poincaré section $\mathcal{P}$ in the energy shell, through an arbitrary point of α. If $\mathcal{P}$ cuts through an encounter, as in figure 1, it must intersect γ in a point close to the reference point on α. Assuming two freedoms we decompose the separation between both points into components s, u along the stable and unstable manifolds [2]. Both s and u must be small, $|u| < c$, $|s| < c$, with c some classically small constant. The components s and u fix the action difference as $\Delta S = su$ and the encounter duration as $t_{\text{enc}} = \frac{1}{\lambda} \ln \frac{c^2}{|su|}$, where λ is the Lyapunov constant [2].

Using ergodicity, we count the encounters within trajectory pairs. The probability density for γ to pierce through $\mathcal{P}$ at a specified time with phase-space separations s and u is uniform and given by the inverse of the volume of the energy shell Ω. To capture all encounters, we integrate that density over (i) the time of piercing on γ and (ii) the time of the reference piercing on α. The probability for $\mathcal{P}$ to cut an encounter is proportional to the duration t_{enc}, which we divide out to get the number of encounters [2]. Changing the integration variables to the loop durations t_1 and t_3, we get the weight function

$$w(s,u) = \int_0^{T_\alpha - t_{\mathrm{enc}}(s,u)} \mathrm{d}t_1 \int_0^{T_\gamma - t_{\mathrm{enc}}(s,u)} \mathrm{d}t_3 \, \frac{1}{\Omega t_{\mathrm{enc}}(s,u)}. \tag{5}$$

Here the upper boundaries, depending on the dwell times T_α, T_γ of α and γ, make sure that the loop durations t_2 and t_4 remain positive. The density $w(s,u)$ is normalized such that $\int \mathrm{d}s\,\mathrm{d}u\, w(s,u)\delta(su - \Delta S)$ is the number density of 2-encounters with fixed action difference ΔS.

To account for all quadruplets of figure 1, we do the sum over β, δ in (2) by integrating with the weight $w(s,u)$,

$$\langle \mathrm{Tr}(tt^\dagger tt^\dagger)\rangle_{2\text{-enc}} = \frac{1}{T_H^2}\left\langle \sum_{\substack{a_1,c_1\\a_2,c_2}} \int \mathrm{d}s\,\mathrm{d}u \sum_{\alpha,\gamma} |A_\alpha|^2 |A_\gamma|^2 w(s,u)\,\mathrm{e}^{\mathrm{i}su/\hbar}\right\rangle, \tag{6}$$

approximating $A_\beta A_\delta \approx A_\alpha A_\gamma$.[3] Now, similarly to (4), we replace the sum over α, γ by an integral over the dwell times or, equivalently, over the loop durations t_2 and t_4. The integrand must be weighted with the probability $\mathrm{e}^{-\frac{N}{T_H}T_{\mathrm{exp}}}$ for both trajectories to remain inside the cavity. Regarding the sum over channels a_1, a_2, c_1, c_2 one might expect a factor $N_1^2 N_2^2$. However, when both the ingoing channels and the outgoing channels coincide, $a_1 = c_1, a_2 = c_2$, each of the two dashed trajectories in figure 1 could be chosen as β or δ, such that the resulting contributions are doubled. Including such combinations for a second time, we get the factor $N_1 N_2 (N_1 N_2 + 1)$. Altogether, we thus find

$$\langle \mathrm{Tr}(tt^\dagger tt^\dagger)\rangle_{2\text{-enc}} = \frac{N_1N_2(N_1N_2+1)}{T_H^2}\left\langle \int_0^\infty \mathrm{d}t_1\,\mathrm{d}t_2\,\mathrm{d}t_3\,\mathrm{d}t_4 \right.$$
$$\left. \times \int \mathrm{d}s\,\mathrm{d}u\,\frac{1}{\Omega t_{\mathrm{enc}}(s,u)}\,\mathrm{e}^{-\frac{N}{T_H}[t_1+t_2+t_3+t_4+t_{\mathrm{enc}}(s,u)]}\,\mathrm{e}^{\frac{\mathrm{i}su}{\hbar}}\right\rangle. \tag{7}$$

The integral factors into four independent integrals over the loop durations, $\int_0^\infty \mathrm{d}t_i\,\mathrm{e}^{-\frac{N}{T_H}t_i} = \frac{T_H}{N}$, and one integral over the separations s, u inside the encounter, $\int \mathrm{d}s\,\mathrm{d}u\,\frac{1}{\Omega t_{\mathrm{enc}}(s,u)}\,\mathrm{e}^{-\frac{N}{T_H}t_{\mathrm{enc}}(s,u)}\,\mathrm{e}^{\mathrm{i}su/\hbar} \xrightarrow{\hbar\to 0} -\frac{N}{T_H^2}$ as shown in [4]. Since all powers of T_H mutually cancel, the following *diagrammatic rule* arises: *each loop gives rise to a factor* $\frac{1}{N}$, *and an encounter contributes a factor* $-N$. The rule yields $-1/N^3$; upon multiplying with the number of possible combinations of channels, we get

$$-\langle \mathrm{Tr}(tt^\dagger tt^\dagger)\rangle_{2\text{-enc}} = \frac{N_1N_2(N_1N_2+1)}{N^3}, \tag{8}$$

i.e., for $N_1, N_2 \gg 1$ the leading shot-noise term in (1).

All orders. To go beyond the leading term, we have to account for all quadruplets of trajectories differing in arbitrarily many encounters, each involving arbitrarily many stretches;

[3] This approximation is justified because the stability amplitudes of α and γ can be approximated as products of contributions from loops and stretches, practically coinciding with those of β and δ.

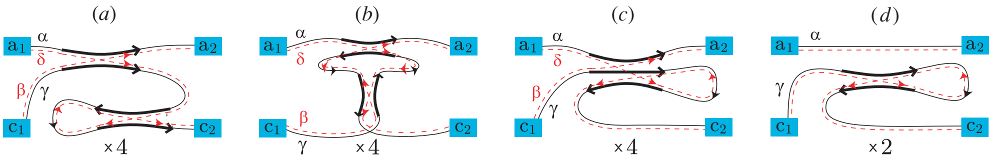

Figure 2. Families of trajectory quadruplets $\alpha, \beta, \gamma, \delta$ responsible for the next-to-leading ('weak localization') contribution to shot noise (drawn similar as in figure 1, but without the cavity). Each picture represents either four or two similar families of quadruplets. Arrows indicate the direction of motion inside the encounters (thick lines), and highlight loops which are traversed in opposite direction by (α, γ) and (β, δ).

figure 2 shows a few examples. In the unitary case we must consider encounters where several stretches of either α or γ, or both, come close in phase space, whereas in the presence of time-reversal invariance the stretches may also be nearly mutually time-reversed.

The contributions of all families of quadruplets obey the above diagrammatic rule. To show this, we describe each l-encounter (encounter of l stretches) by $l-1$ pairs of coordinates $s_j, u_j, j = 1, \ldots, l-1$ [2] measuring the separations of $l-1$ stretches from one reference stretch. These coordinates determine both the duration of each encounter stretch and its contribution $\sum_{j=1}^{l-1} s_j u_j$ to the action difference. The analog of the density $w(s, u)$ in (5) obtains a factor $\frac{1}{\Omega^{l-1} t_{\mathrm{enc}}(s,u)}$ from each l-encounter. The resulting product must be integrated over the durations of all loops, with integration over the final loops of α and γ coming into play through the summation over α and γ as in (7). Finally, the contribution of each family factors into 'loop' and 'encounter' integrals similar to those in (7). After cancellation of all powers of T_H, the diagrammatic rule comes about, with a factor $\frac{1}{N}$ from each loop and a factor $-N$ from each encounter.

Again, we have to multiply the result with the number of possible combinations of channels. Two cases must be distinguished. First, let us consider trajectory quadruplets as in figures 2(*a*)–(*c*) where, similarly to figure 1, the partner trajectories β and δ connect the initial point of α to the final point of γ, and the initial point of γ to the final point of α. Such quadruplets will be called *x-quadruplets*. For them, the channels a_1, c_1, a_2, and c_2 may be chosen arbitrarily and allow for $N_1 N_2 (N_1 N_2 + 1)$ combinations.

In contrast, figure 2(*d*) depicts a quadruplet where the partner trajectories connect the initial point of α to the final point of α, and the initial point of γ to the final point of γ, similarly to the diagonal contribution. We speak of a *d-quadruplet* then. The partner trajectories now connect the leads as $a_1 \to a_2, c_1 \to c_2$. Since for shot noise we need partner trajectories $\beta\,(a_1 \to c_2)$, $\delta\,(c_1 \to a_2)$, d-quadruplets contribute only if either the two ingoing channels, or the two outgoing channels (and thus the corresponding angles of incidence) coincide. As for the diagonal contribution, we thus obtain $N_1^2 N_2 + N_1 N_2^2 = N N_1 N_2$ possible combinations.

Our diagrammatic rules determine $\langle \mathrm{Tr}(tt^\dagger tt^\dagger)\rangle$ as

$$\langle \mathrm{Tr}(tt^\dagger tt^\dagger)\rangle = \frac{N_1 N_2 (N_1 N_2 + 1)}{N^2} \sum_{m=1}^{\infty} \frac{x_m}{N^m} + \frac{N_1 N_2}{N} \left\{ 1 + \sum_{m=1}^{\infty} \frac{d_m}{N^m} \right\}. \tag{9}$$

Towards explaining x_m and d_m we denote the number of encounters in a quadruplet by V and the total number of the encounter stretches by L. Then x_m is the number of families of x-quadruplets with $m = L - V$ and even V, minus the number of corresponding families with odd V; d_m is the analogous number of d-families. We note that the contribution of each family is proportional to $\frac{1}{N^{m+2}}$ rather than $\frac{1}{N^m}$, since there are two more loops than encounter stretches.

The leading contribution to shot noise originates from the family of figure 1; it gives $x_1 = -1$. For the next-to-leading term, we have to consider x-quadruplets with two 2-encounters or one 3-encounter, contributing to x_2 and depicted in figures 2(a)–(c), and d-quadruplets which are related to a single 2-encounter and contribute to d_1 (figure 2(d)). All quadruplets in figure 2 involve mutually time-reversed loops and can exist in the orthogonal case only (thus $x_2 = d_1 = 0$ in the unitary case). Note that if we interchange the two leads, or the pairs (α, γ) and (β, δ), or the trajectories α and γ, each family of quadruplets will be either left topologically invariant or turned into an equivalent family making the same contribution to the shot noise. Only one representative of each such 'symmetry multiplet' is shown in figure 2, with the number of equivalent families indicated by a multiplier. The sum of all contributions gives $x_2 = 4$, $d_1 = -2$, and $\langle \mathrm{Tr}(tt^\dagger tt^\dagger)\rangle = \frac{N_1N_2}{N} - \frac{N_1^2N_2^2}{N^3} + 4\frac{N_1^2N_2^2}{N^4} - 2\frac{N_1N_2}{N^2} + \mathcal{O}\left(\frac{1}{N}\right)$. Together with $\langle \mathrm{Tr}(tt^\dagger)\rangle = \frac{N_1N_2}{N} - \frac{N_1N_2}{N^2} + \mathcal{O}\left(\frac{1}{N}\right)$, we recover the second term in (1).

For higher orders in $\frac{1}{N}$, we must collect all families of trajectory quadruplets. We had previously established a method for counting families of *pairs* of *periodic orbits* differing in encounters, based on permutation theory [2]. The families of orbit pairs thus obtained can be turned into the families of trajectory quadruplets needed now, simply by cutting each pair twice, inside loops. One can show that if one and one only of the loops cut is traversed in the opposite sense in the orbits of the pair, the resulting quadruplet is of type x, and otherwise of type d. Using this method we obtain

$$x_m = \begin{cases} \dfrac{(-1)^m - 1}{2} & \text{unitary} \\ (-1)^m \dfrac{3^m - 1}{2} & \text{orthogonal} \end{cases}$$

$$d_m = \begin{cases} \dfrac{(-1)^m + 1}{2} & \text{unitary} \\ (-1)^m \dfrac{3^m + 1}{2} & \text{orthogonal.} \end{cases} \tag{10}$$

The proof, based on a recursion derived in [2], will be given elsewhere. Summing over m, we get the shot noise

$$P = \begin{cases} \dfrac{N_1^2N_2^2}{N(N^2-1)} & \text{unitary} \\ \dfrac{N_1(N_1+1)N_2(N_2+1)}{N(N+1)(N+3)} & \text{orthogonal,} \end{cases} \tag{11}$$

valid to all orders in $\frac{1}{N}$ and thus also for a few channels.

Weak magnetic field. In the presence of a weak magnetic field B, the power of shot noise must interpolate between the orthogonal and unitary cases. A weak field increases the action of each trajectory [4, 14] by the line integral $\frac{e}{c}\int \mathbf{A}\cdot \mathrm{d}\mathbf{q}$ of the vector potential $\mathbf{A}$. Since that increment changes sign under time reversal a net contribution survives from all loops and encounter stretches changing direction in (β, δ) relative to (α, γ). Our above diagrammatic rule is thus modified: each loop changing direction contributes $\frac{1}{N(1+\xi)}$ with $\xi \propto B^2$; each encounter contributes $-N(1 + \xi\mu^2)$ with μ being the number of its stretches changing direction [4].

The leading contribution to shot noise, from quadruplets as in figure 1, remains unaffected by the magnetic field. However, all quadruplets responsible for the next-to-leading term obtain a Lorentzian factor $\frac{1}{1+\xi}$. (In figures 2(a), (c), and (d) one loop is traversed in time-reversed sense and $\mu = 0$, whereas in figure 2(*b*) two loops are time-reversed and one encounter has $\mu = 1$.) We thus predict

$$P = \frac{N_1^2 N_2^2}{N^3} + \frac{N_1 N_2 (N_1 - N_2)^2}{N^4(1+\xi)} + \mathcal{O}\left(\frac{1}{N}\right), \tag{12}$$

in accordance with (1) for the limits $\xi \to 0$ and $\xi \to \infty$. If $N_1 = N_2 = N/2$ the second term in (12) vanishes, and quadruplets involving more encounter stretches yield

$$P = \frac{N}{16} + \frac{1}{N} \frac{1 + 8\xi + 4\xi^2 + 4\xi^3 + \xi^4}{16(1+\xi)^4} + \mathcal{O}\left(\frac{1}{N^2}\right); \tag{13}$$

the extension to $\mathcal{O}(\frac{1}{N^6})$ will be given elsewhere.

Outlook. The semiclassical approach to transport opens a large field of experimentally relevant applications. For instance, we have checked the trajectory quadruplets employed here to also yield the universal conductance variance as well as the covariance of the conductance at two different energies, both within and in between the orthogonal and unitary symmetry classes.

Acknowledgments

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